An Ardteistiméireacht
Ardleibhéal Páipéar 1
Uimhreacha Coimpléascacha

2017 C2

2016 C1

2015 C4

2014 C2

SEC sampla
2014 C1

2013 C1

2012 C3

2011 C2

www.mathsplus.ie

An Chomhairle um Oideachas Gaeltachta & Gaelscolaíochta

Páipéar 1 Ceist 2
2017

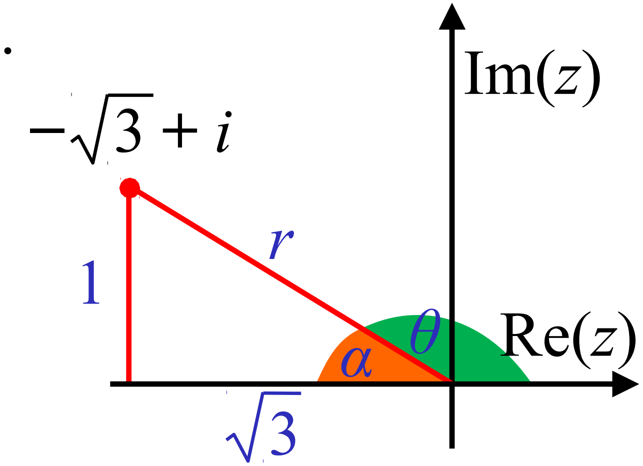
25 marc

$a^2+2ab+b^2=(a+b)^2$

An Chomhairle um Oideachas Gaeltachta & Gaelscolaíochta

$z = -\sqrt{3} + i$ áit a bhfuil $i^2 = -1$.

(a) Úsáid Teoirim de Moivre chun z^4 a scríobh san fhoirm $a + b\sqrt{c}\, i$, áit a bhfuil a, b, agus $c \in \mathbb{Z}$.



$\tan \alpha = \frac{1}{\sqrt{3}} \Rightarrow \alpha = 30° \Rightarrow \theta = 150°$

$r = \sqrt{1^2 + \sqrt{3}^2} = \sqrt{4} = 2$

Teoirim de Moivre

$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

$$\left(-\sqrt{3} + i\right)^4 = \left(2(\cos 150° + i\sin 150°)\right)^4$$

$$= 2^4(\cos(4\times 150°) + i\sin(4\times 150°))$$

$$= 16(\cos 600° + i\sin 600°)$$

$$= 16\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$

$$= -8 - 8\sqrt{3}\,i$$

15

(b) Is uimhir choimpléascach é w sa chaoi go bhfuil $|w| = 3$ agus déanann w uillinn 30° le treo deimhneach na haise réadaí. Má tá $t = zw$, scríobh t san fhoirm is simplí.

$$w = r(\cos\theta + i\sin\theta) \qquad r = 3,\ \theta = 30^\circ$$

$$= 3(\cos 30^\circ + i\sin 30^\circ)$$

$$= 3\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \frac{3\sqrt{3}}{2} + \frac{3}{2}i$$

$$t = zw$$

$$= \left(-\sqrt{3} + i\right)\left(\frac{3\sqrt{3}}{2} + \frac{3}{2}i\right)$$

$$= -\frac{3(3)}{2} - \cancel{\frac{3\sqrt{3}}{2}}i + \cancel{\frac{3\sqrt{3}}{2}}i + \frac{3}{2}\cancel{i^2}(-1)$$

$$= -\frac{12}{2}$$

$$= -6$$

10

Páipéar 1 Ceist 1
2016

25 marc

$a^2+2ab+b^2=(a+b)^2$

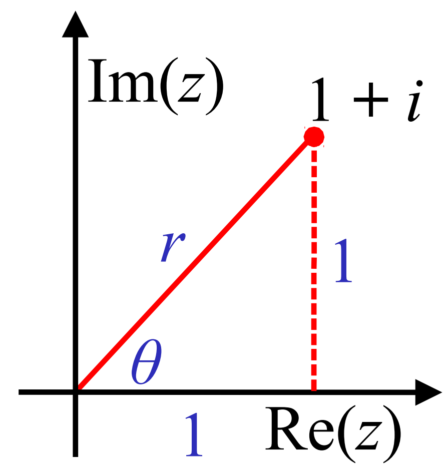
An Chomhairle um Oideachas Gaeltachta & Gaelscolaíochta

(a) Tá $(-4+3i)$ ina fhréamh den chothromóid $az^2 + bz + c = 0$, áit a bhfuil $a, b, c \in \mathbb{R}$, agus $i^2 = -1$. Scríobh an fhréamh eile.

is fréamh é $(-4-3i)$ freisin

5

(b) Bain úsáid as Teoirim De Moivre chun $(1+i)^8$ a shloinneadh san fhoirm is simplí.



$$\tan\theta = \frac{1}{1} \Rightarrow \theta = \frac{\pi}{4} \qquad r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$(1+i)^8 = \left\{\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right\}^8$$

Teoirim de Moivre

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

$$(1+i)^8 = \sqrt{2}^{\,8}\left(\cos(8)\frac{\pi}{4} + i\sin(8)\frac{\pi}{4}\right) = 16\left(\cos 2\pi + i\sin 2\pi\right)$$

$$= 16(1+0i)$$

$$= 16$$

10

(c) Tá $(1 + i)$ ina fhréamh den chothromóid

$$z^2 + (-2 + i)\,z + 3 - i = 0.$$

Faigh an fhréamh eile san fhoirm $m + ni$, áit a bhfuil $m, n \in \mathbb{R}$, agus $i^2 = -1$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$, $b = -2 + i$ agus $c = 3 - i$

$$z = \frac{-(-2 + i) \pm \sqrt{(-2 + i)^2 - 4(1)(3 - i)}}{2(1)}$$

$$= \frac{2 - i \pm \sqrt{4 - \cancel{4i} + i^2 - 12 + \cancel{4i}}}{2(1)}$$

$$= \frac{2 - i \pm \sqrt{-9}}{2}$$

$$= \frac{2 - i \pm 3i}{2} = 1 + i \text{ nó } 1 - 2i$$

10

Páipéar 1 Ceist 4
2015

25 marc

An Chomhairle um Oideachas Gaeltachta & Gaelscolaíochta

(a) Is uimhreacha coimpléascacha iad z_1, z_2 agus z_3 sa chaoi go bhfuil $\dfrac{2}{z_1} = \dfrac{1}{z_2} + \dfrac{1}{z_3}$, $z_2 = 2 + 3i$ agus $z_3 = 3 - 2i$, áit a bhfuil $i^2 = -1$.

Scríobh z_1 san fhoirm $a + bi$, áit a bhfuil $a, b \in \mathbb{R}$.

$$\frac{2}{z_1} = \frac{1}{z_2} + \frac{1}{z_3} = \frac{1}{2+3i} + \frac{1}{3-2i} = \frac{3-2i+2+3i}{(2+3i)(3-2i)}$$

$$\frac{2}{z_1} = \frac{5+i}{6-4i+9i+6(-1)}$$

Bailigh téarmaí atá cosúil le chéile

$$\frac{2}{z_1} = \frac{(24+10i)}{12+5i} \times \frac{5-i}{5-i}$$

Iolraigh faoi 2

$$z_1 = \frac{120-24i+50i+10(-1)}{25-\cancel{5i}+\cancel{5i}+1}$$

$$= \frac{130+26i}{26}$$

Deighilt ag 26

$$= 5 + i$$

15

Chun uimhir choimpléascach a roinnt ar uimhir choimpléascach eile, caithfimid an t-ainmneoir a athrú go réaduimhir trí chomhchuingeach coimpléascach a úsáid

(b) Bíodh ω ina uimhir choimpléascach, áit a bhfuil $\omega^n = 1$, $\omega \neq 1$, agus $S = 1 + \omega + \omega^2 + \ldots + \omega^{n-1}$. Bain úsáid as an bhfoirmle le haghaidh suim sraithe iolraíche críochta chun luach S a scríobh san fhoirm is simplí.

$$S = 1 + \omega + \omega^2 + \ldots + \omega^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \qquad a = 1,\ r = \omega$$

$$S = \frac{1(1-\omega^n)}{1-\omega}$$

$$= \frac{1(1-1)}{1-\omega}$$

$$= \frac{1(0)}{1-\omega}$$

$$= 0$$

10

Páipéar 1 Ceist 2
2014

25 marc

Bíodh $z_1 = 1 - 2i$, áit a bhfuil $i^2 = -1$

(a) Tá an uimhir choimpléascach z_1 ina fréamh den chothromóid $2z^3 - 7z^2 + 16z - 15 = 0$.
Faigh an dá fhréamh eile atá ag an gcothromóid.

$z_1 = 1 - 2i$ fréamh $\Rightarrow$ $\overline{z}_1 = 1 + 2i$ fréamh

$(z - 1 + 2i)(z - 1 - 2i) = z^2 - 2z + 5$

$$2z - 3 \Rightarrow z = \frac{3}{2}$$

$$
\begin{array}{r|l}
 & 2z - 3 \\
\hline
z^2 - 2z + 5 & \cancel{2z^3} - 7z^2 + 16z - 15 \\
 & -\cancel{2z^3} + 4z^2 - 10z \\
\hline
 & -3z^2 + 6z - 15 \\
 & -3z^2 + 6z - 15 \\
\hline
 & 0
\end{array}
$$

Fréamhacha eile: $1 + 2i$ agus $\frac{3}{2}$

5

Bíodh $z_1 = 1 - 2i$, áit a bhfuil $i^2 = -1$

(b) **(i)** Bíodh $w = \bar{z}_1 . z_1$, áit a bhfuil $\bar{z}_1$ ina chomhchuingeach de z_1.

Breac z_1, $\bar{z}_1$ agus w ar an léaráid Argand agus lipéadaigh gach pointe.

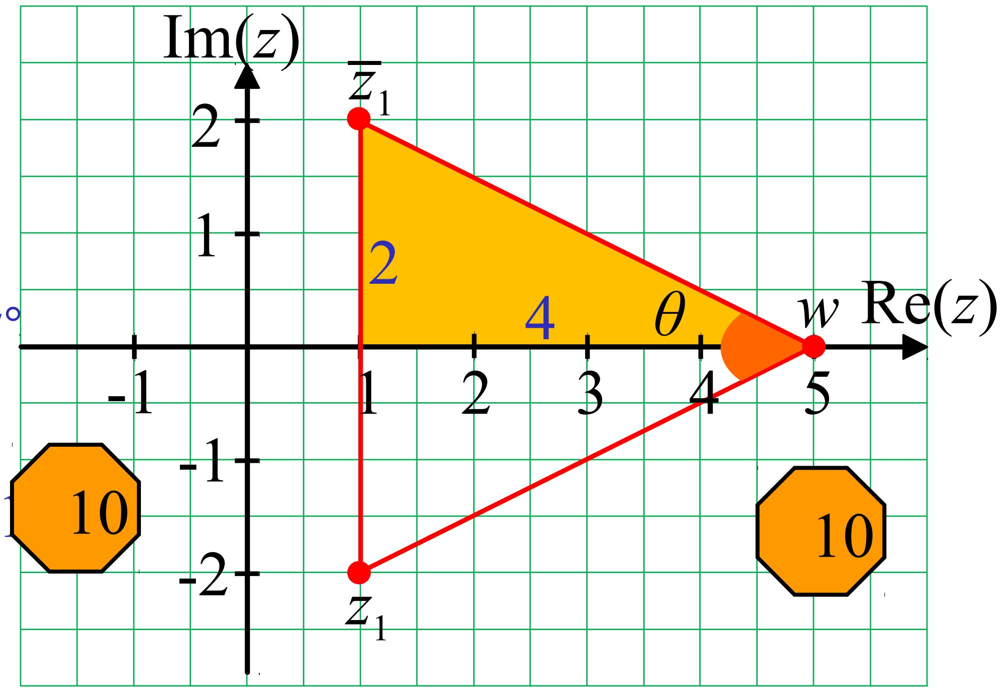
$\bar{z}_1 = 1 + 2i$

$\tan\theta = \dfrac{2}{4}$

$\theta = 26 \cdot 57°$

$\angle \bar{z}_1 w z_1 = 2\theta$

$= 53°$



$w = z_1 . \bar{z}_1 = (1 - 2i)(1 + 2i) = 1 - 2i + 2i - 4i^2 = 5$

(ii) Faigh méid na géaruillinne,, $\bar{z}_1 w z_1$, a dhéantar nuair a cheanglaítear $\bar{z}_1$ go w to z_1 ar an léaráid thuas.

Bíodh do fhreagra ceart go dtí an chéim is gaire.

Páipéar 1 Ceist 1
2014 Sampla SEC

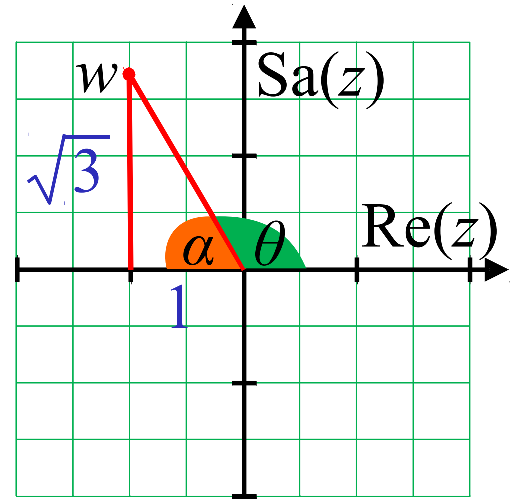
25 marc

$a^2+2ab+b^2=(a+b)^2$

An Chomhairle um Oideachas Gaeltachta & Gaelscolaíochta

(a) Is uimhir choimpléascach í $w = -1 + \sqrt{3}\,i$,áit a bhfuil $i^2 = -1$.

(i) Scríobh w san fhoirm pholach.



$$\tan\alpha = \frac{\sqrt{3}}{1}$$

$$\alpha = \frac{\pi}{3}$$

$$\theta = \frac{2\pi}{3}$$

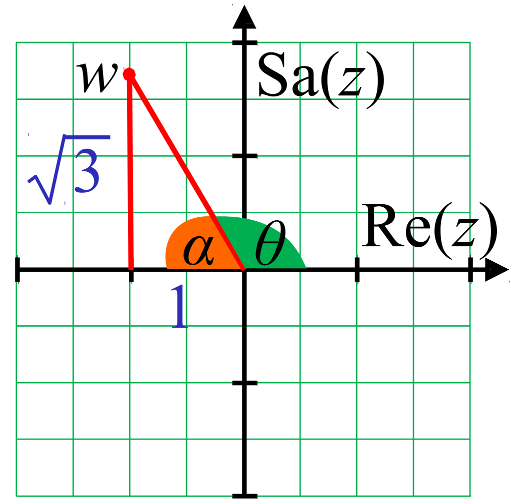
$$r = \sqrt{1^2 + \sqrt{3}^{\,2}}$$

$$r = \sqrt{4} = 2$$

$$w = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

(a) Is uimhir choimpléascach í $w = -1 + \sqrt{3}\,i$, áit a bhfuil $i^2 = -1$.

(ii) Bain úsáid as teoirim De Moivre chun an chothromóid $z^2 = -1 + \sqrt{3}\,i$ a réiteach. Tabhair do fhreagra(í) i bhfoirm dhronuilleogach.



$$z^2 = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$ cuid **(i)**

$$z = 2^{\frac{1}{2}}\left(\cos\left(\frac{2\pi}{3} + 2n\pi\right) + i\sin\left(\frac{2\pi}{3} + 2n\pi\right)\right)^{\frac{1}{2}}$$

$$z = \sqrt{2}\left(\cos\left(\frac{\pi}{3} + n\pi\right) + i\sin\left(\frac{\pi}{3} + n\pi\right)\right)$$

Teoirim de Moivre

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

Bíodh $n = 0$

$$z_1 = \sqrt{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$z_1 = \sqrt{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

Bíodh $n = 1$

$$z_2 = \sqrt{2}\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$$

$$z_2 = \sqrt{2}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$

(b) Taispeántar ceithre uimhir choimpléascacha z_1 , z_2 , z_3 agus z_4 ar léaráid Argand. Sásaíonn siad na coinníollacha seo a leanas:

$z_2 = iz_1$ rothlú 90° ó

$z_3 = kz_1$, áit a bhfuil $k \in \mathbb{R}$ éifeacht rite ó

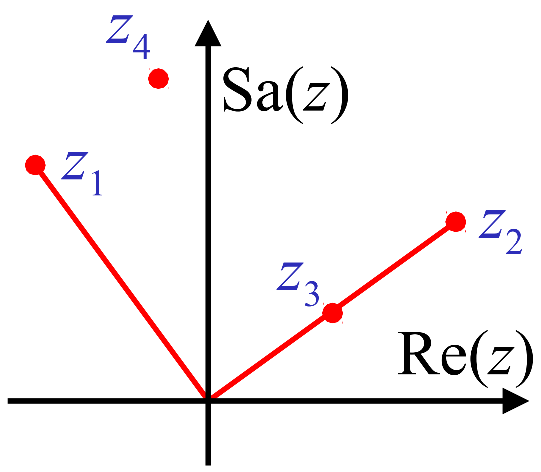
$z_4 = z_2 + z_3$.

Úsáidtear an scála céanna ar an dá ais.

(i) Cuir na huimhreacha in iúl le lipéad ar na pointí ar an léaráid.

(ii) Scríobh síos garluach k. $k = 0 \cdot 5$

Freagra:____________



Páipéar 1 Ceist 1
2013

25 marc

Is uimhir choimpléascach í $z = \dfrac{4}{1+\sqrt{3}\,i}$ áit a bhfuil $i^2 = -1$.

(a) Fíoraigh gur féidir z a scríobh mar $\sqrt{1} - 3i$.

$$z = \frac{4}{1+\sqrt{3}\,i} \times \frac{1-\sqrt{3}\,i}{1-\sqrt{3}\,i} = \frac{4-4\sqrt{3}\,i}{1-\cancel{\sqrt{3}\,i}+\cancel{\sqrt{3}\,i}+3i^2(-1)}$$

$$= \frac{\cancel{4}-\cancel{4}\sqrt{3}\,i}{\cancel{4}}$$ Deighilt ag 4

$$= 1-\sqrt{3}\,i$$

10

Chun uimhir choimpléascach a roinnt ar uimhir choimpléascach eile, caithfimid an t-ainmneoir a athrú go réaduimhir trí chomhchuingeach coimpléascach a úsáid

(b) Breac z ar léaráid Argand agus scríobh z san fhoirm pholach.

$$z = 1 - \sqrt{3}i$$

$$\tan \alpha = \frac{\sqrt{3}}{1}$$

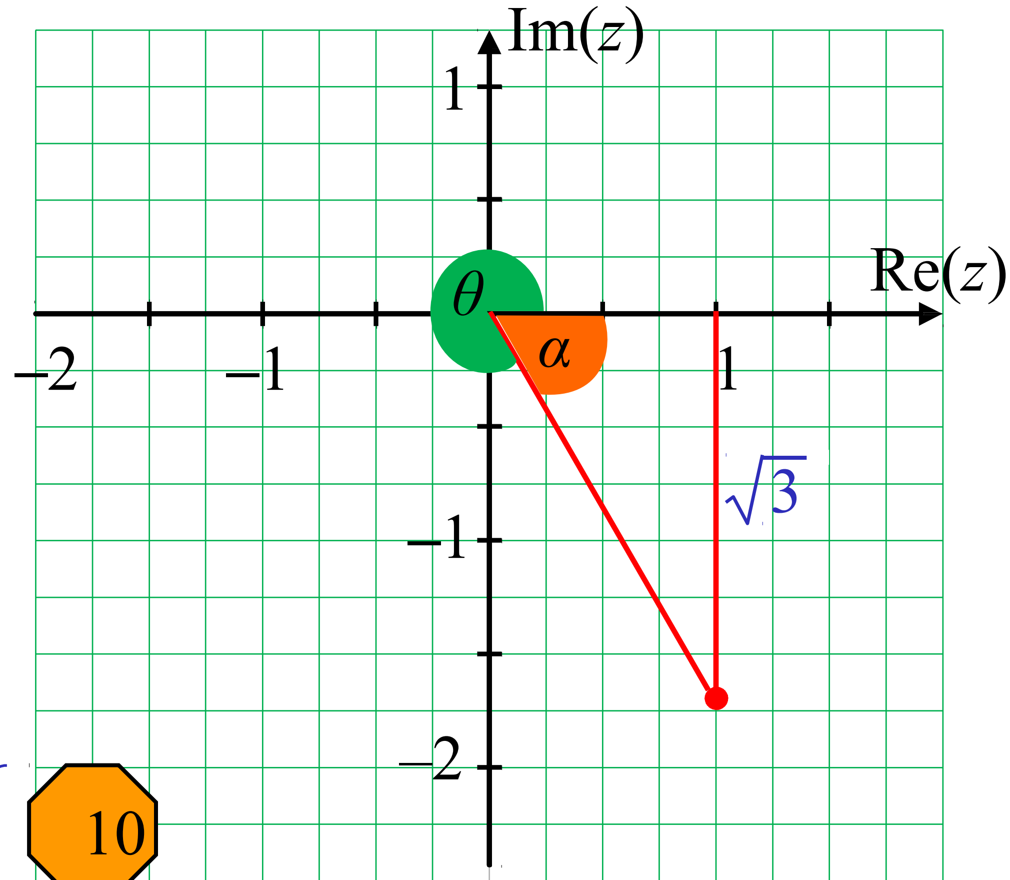
$$\alpha = \frac{\pi}{3}$$

$$\theta = \frac{5\pi}{3}$$

$$r = \sqrt{1^2 + \sqrt{3}^2}$$

$$r = \sqrt{4} = 2$$

$$z = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$$



10

(c) Bain úsáid as teoirim De Moivre chun a thaispeáint go bhfuil

$$z^{10} = -2^9\left(1 - \sqrt{3}i\right).$$

$$z^{10} = 2^{10}\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)^{10}$$

Teoirim de Moivre

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

$$z^{10} = 2^{10}\left(\cos\frac{50\pi}{3} + i\sin\frac{50\pi}{3}\right)$$

$$= 2^{10}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$= 2^{10}\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$

$$= -2^9\left(1 - \sqrt{3}i\right)$$

5

Páipéar 1 Ceist 3
2012

25 marc

$a^2+2ab+b^2=(a+b)^2$

An Chomhairle um Oideachas Gaeltachta & Gaelscolaíochta

Tá modal $5\frac{1}{16}$ agus argóint $\frac{4\pi}{9}$ ag an uimhir choimpléascach z.

(a) Faigh, san fhoirm pholach, na ceithre cheathrú fréamh choimpléascacha atá ag z. (Is é sin, faigh na ceithre luach ar w a fhágann go bhfuil $w^4 = z$.)

$$z = \frac{81}{16}\left(\cos\left(\frac{4\pi}{9} + 2n\pi\right) + i\sin\left(\frac{4\pi}{9} + 2n\pi\right)\right)$$

$$w^4 = \frac{81}{16}\left(\cos\left(\frac{4\pi}{9} + 2n\pi\right) + i\sin\left(\frac{4\pi}{9} + 2n\pi\right)\right)$$

$$w = \left(\frac{81}{16}\right)^{\frac{1}{4}}\left(\cos\left(\frac{4\pi}{9} + 2n\pi\right) + i\sin\left(\frac{4\pi}{9} + 2n\pi\right)\right)^{\frac{1}{4}}$$

$$w = \frac{3}{2}\left(\cos\left(\frac{\pi}{9} + \frac{n\pi}{2}\right) + i\sin\left(\frac{\pi}{9} + \frac{n\pi}{2}\right)\right)$$

Teoirim de Moivre

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

Tá modal $5\frac{1}{16}$ agus argóint $\frac{4\pi}{9}$ ag an uimhir choimpléascach z.

(a) Faigh, san fhoirm pholach, na ceithre cheathrú fréamh choimpléascacha atá ag z. (Is é sin, faigh na ceithre luach ar w a fhágann go bhfuil $w^4 = z$.)

$$w = \frac{3}{2}\left(\cos\left(\frac{\pi}{9} + \frac{n\pi}{2}\right) + i\sin\left(\frac{\pi}{9} + \frac{n\pi}{2}\right)\right)$$

20

Bíodh $n = 0$

$$w_0 = \frac{3}{2}\left(\cos\frac{\pi}{9} + i\sin\frac{\pi}{9}\right)$$

Bíodh $n = 1$

$$w_1 = \frac{3}{2}\left(\cos\left(\frac{\pi}{9} + \frac{1\pi}{2}\right) + i\sin\left(\frac{\pi}{9} + \frac{1\pi}{2}\right)\right)$$

$$w_1 = \frac{3}{2}\left(\cos\frac{11\pi}{18} + i\sin\frac{11\pi}{18}\right)$$

Bíodh $n = 2$

$$w_2 = \frac{3}{2}\left(\cos\left(\frac{\pi}{9} + \frac{2\pi}{2}\right) + i\sin\left(\frac{\pi}{9} + \frac{2\pi}{2}\right)\right)$$

$$w_2 = \frac{3}{2}\left(\cos\frac{10\pi}{9} + i\sin\frac{10\pi}{9}\right)$$

Bíodh $n = 3$

$$w_3 = \frac{3}{2}\left(\cos\left(\frac{\pi}{9} + \frac{3\pi}{2}\right) + i\sin\left(\frac{\pi}{9} + \frac{3\pi}{2}\right)\right)$$

$$w_3 = \frac{3}{2}\left(\cos\frac{29\pi}{18} + i\sin\frac{29\pi}{18}\right)$$

Tá modal $5\frac{1}{16}$ agus argóint $\frac{4\pi}{9}$ ag an uimhir choimpléascach z.

(b) Tá z marcáilte ar an léaráid Argand thíos. Ar an léaráid chéanna, taispeáin na ceithre fhreagra ar chuid **(a)**.

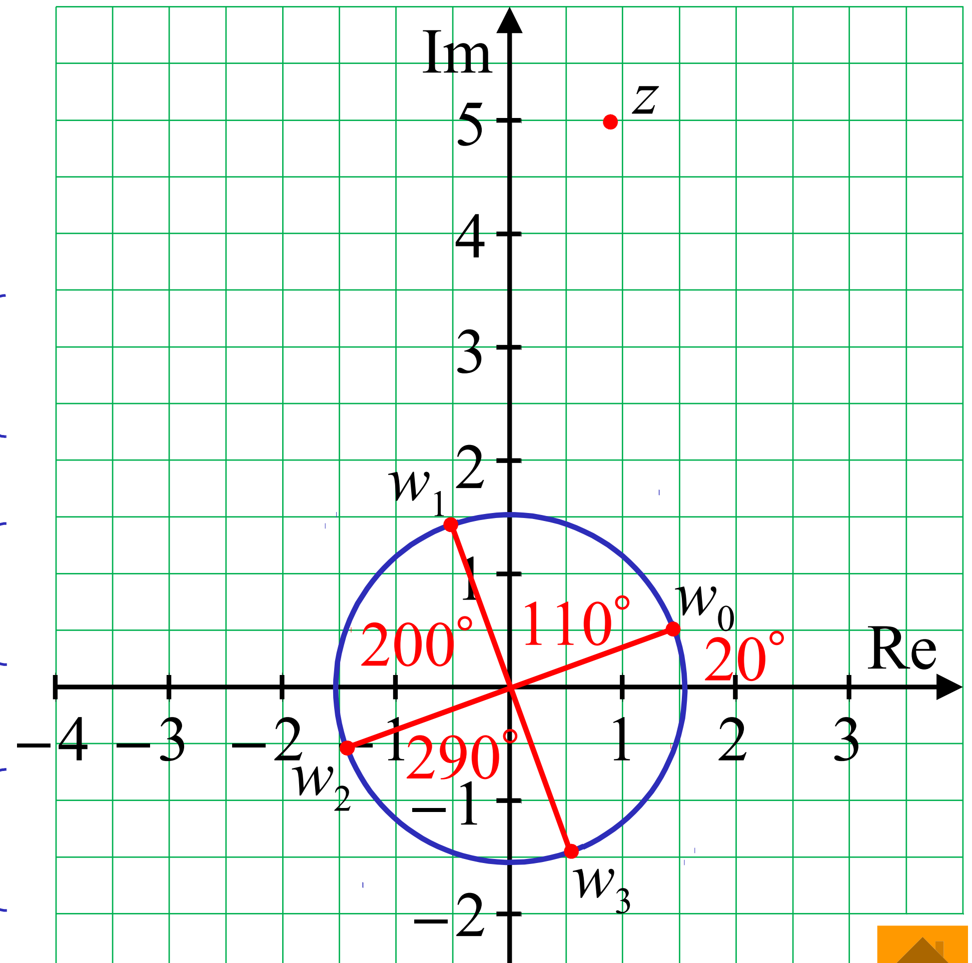
5

$$w_0 = \frac{3}{2}\left(\cos\frac{\pi}{9} + i\sin\frac{\pi}{9}\right)$$

$$w_1 = \frac{3}{2}\left(\cos\frac{11\pi}{18} + i\sin\frac{11\pi}{18}\right)$$

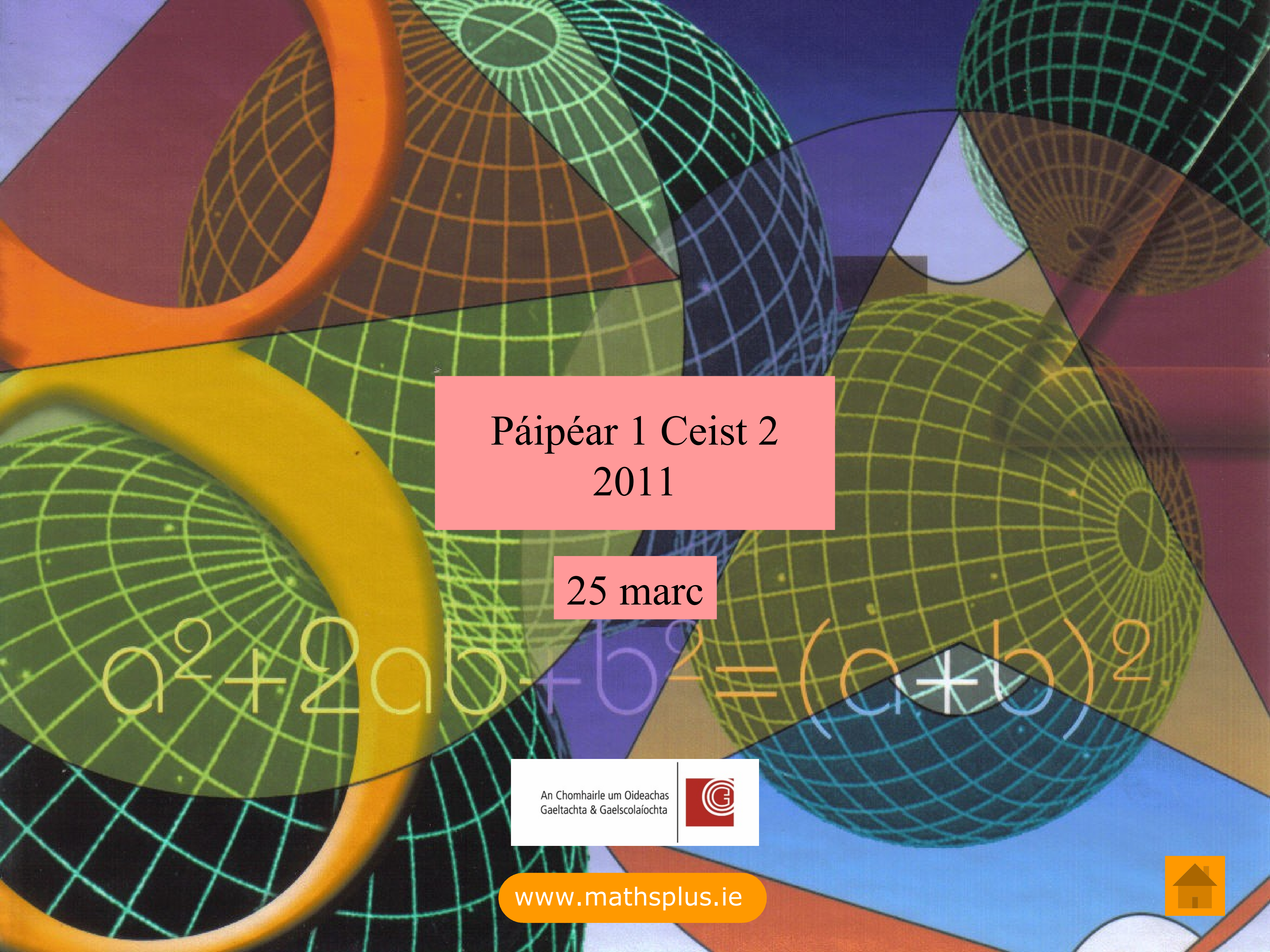
$$w_2 = \frac{3}{2}\left(\cos\frac{10\pi}{9} + i\sin\frac{10\pi}{9}\right)$$

$$w_3 = \frac{3}{2}\left(\cos\frac{29\pi}{18} + i\sin\frac{29\pi}{18}\right)$$



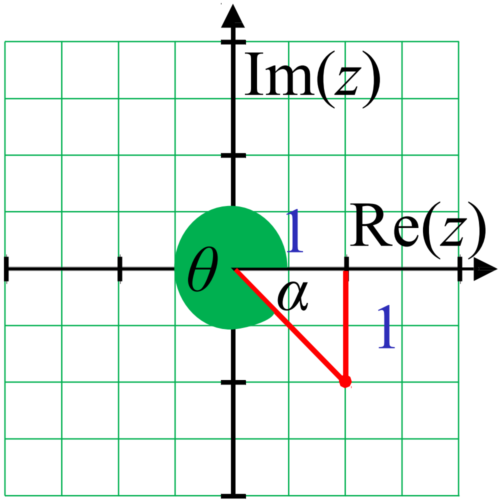
Páipéar 1 Ceist 2
2011

25 marc



An Chomhairle um Oideachas Gaeltachta & Gaelscolaíochta

(a) **(i)** Scríobh an uimhir choimpléascach $1-i$ san fhoirm pholach.



$$\tan \alpha = 1$$

$$\alpha = \frac{\pi}{4}$$

$$\theta = \frac{7\pi}{4}$$

$$r = \sqrt{1^2 + 1^2}$$

$$r = \sqrt{2}$$

$$1 - i = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

15

(a) **(i)** Scríobh an uimhir choimpléascach $1-i$ san fhoirm pholach.

(ii) Bain úsáid as teoirim De Moivre chun $(1-i)^9$, a luacháil, agus tabhair do fhreagra i bhfoirm dhronuilleogach.

$$(1-i)^9 = \sqrt{2}^{\,9}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)^9$$

Teoirim de Moivre

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

$$(1-i)^9 = 16\sqrt{2}\left(\cos\frac{63\pi}{4} + i\sin\frac{63\pi}{4}\right)$$

$$= 16\sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$$

$$= 16 - 16i$$

5

(b) Tá modal níos mó ná 1 ag uimhir choimpléascach z. Taispeántar ar an léaráid Argand na trí uimhir z, z^2 , agus z^3. Tá ceann díobh ar an ais shamhailteach, mar a thaispeántar.

(i) Lipéadaigh na pointí ar an léaráid chun a thaispeáint cé na huimhreacha a bhfreagraíonn na pointí dóibh.

(ii) Faigh θ, argóint z.

$$z = r(\cos\theta + i\sin\theta)$$

$$z^3 = r^3(\cos 3\theta + i\sin 3\theta)$$

$$\Rightarrow 3\theta = 90^\circ \Rightarrow \theta = 30^\circ$$

z sa 2ú ceathrú

$$\Rightarrow \theta = 150^\circ$$

5